

The Effect of a Central Supermassive Black Hole on the Gas Fuelling

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ABSTRACT

When a supermassive black hole exists in the centre of a galaxy, an additional inner Lindblad resonance (ILR) exists inside the usual ILRs. We study gas dynamics in a weakly barred potential with a central supermassive black hole by using 2D numerical simulations, and we investigate the effect of the additional ILR on fuelling gas into nuclear starburst regions or AGNs. Our numerical results show that strong trailing spiral shocks are formed at the resonance region, and the gas in the shock region is rapidly fuelled into a central region and make a nuclear gas ring. As a result, a large amount of gas is concentrated in the nuclear region beyond the ILR in a dynamical time scale.

Key words: galaxies: nuclei – galaxies: starburst – ISM: kinematics and dynamics – methods: numerical.

1 INTRODUCTION

Nuclear activities in galaxies, such as nuclear starbursts or AGNs, are supposed to be induced by gas fuelling into nuclear regions of galaxies. Observational studies have suggested that non-axisymmetric gravitational potential caused by the stellar bar or galaxy-galaxy interactions is a convincing mechanism for triggering the gas fuelling (Phinney 1994). This is because the non-axisymmetric potential can remove the angular momentum of gas, and a large amount of gas can fall toward the centre in a dynamical time scale. However, numerical simulations have shown that the gas tends to form an oval ring-like structure near the inner Lindblad resonance (ILR) (Elmegreen 1994 and references therein). It is also shown that the bar can not force the gas to accrete toward the galactic centre beyond the ILR. As a mechanism to overcome the ILR barrier, the double barred structure (Friedli & Martinet 1993), or the self-gravity of gas (Wada & Habe 1992, 1995; Elmegreen 1994) are proposed.

For an alternative mechanism to fuel the gas to the galactic centre we investigate the effect of a central supermassive black hole (SBH). Recent observations suggest that some galaxies have central SBHs (Ford et al. 1994; Harms et al. 1994; Miyoshi et al. 1995). If the central SBH exists, an additional ILR (hereafter a nuclear Lindblad resonance (NLR)) appears inside of the usual ILRs. Therefore,

the stellar and gas dynamics in the resonant region are affected by the NLR, and the gas fuelling into the inside of the usual ILRs caused by the NLR is expected. Hasan & Norman (1990) investigated the orbits of a star in a barred galaxy with a central SBH and showed that the stochastic regions appear as the central mass is increased, and the orbits sustaining the bar are dissolved. Pfenniger & Norman (1990) showed that the dissipation is enhanced in the resonance region and that the gas inflow is boosted inside the ILRs (including the NLR). However, since Pfenniger & Norman (1990) used weakly dissipative single particles to represent idealised gas clouds, the effects of the NLR on the dynamics of the actual interstellar matter is not still clear.

In this paper, we investigate the dynamics of a non-self-gravitating gas disc near the NLR region of a weakly barred galaxy with the central SBH by using the smoothed particle hydrodynamics (SPH) method. Our aim is to clarify a role of the new resonance caused by the central SBH on triggering the gas fuelling for the galactic centre.

In section 2, we present models of galaxies with a central SBH and a gas disc, and a numerical method. Numerical results are presented in section 3. In section 4, we summarise our results and discuss the mechanism of gas fuelling by the NLR and the observations related to our numerical results and the effects of the self-gravity of gas.

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2 MODELS

2.1 Model galaxy

The numerical method and models are based on Wada & Habe (1992). We assume that the model galaxy is composed of a stellar disc, a bulge, a weak stellar bar, and a supermassive black hole (SBH) at the centre of the galaxy. The weak stellar bar is treated as an external fixed potential. We do not consider a dark halo component, because we are interested in gas dynamics in the inner region of the galaxy.

Axisymmetric potential of the galaxy is assumed to be the Toomre disc (Toomre 1963) :

$$\Phi_{\text{axi}}(R) = -\frac{c^2}{a} \frac{1}{(R^2 + a^2)^{1/2}}, \quad (1)$$

where a is a core radius and c is given as $c = v_{\text{max}}(27/4)^{1/4}a$, and v_{max} is a maximum rotational velocity in this potential.

We assume the barred potential :

$$\Phi_{\text{bar}}(R, \theta) = \epsilon(R)\Phi_{\text{axi}} \cos 2\theta, \quad (2)$$

where $\epsilon(R)$ is given by

$$\epsilon(R) = \epsilon_0 \frac{aR^2}{(R^2 + a^2)^{3/2}}, \quad (3)$$

and ϵ_0 is a parameter which represents the strength of the bar to the disc component.

We assume the potential of the SBH :

$$\Phi_{\text{BH}}(R) = -f_{\text{BH}} \frac{c^2}{a} \frac{1}{(R^2 + a_{\text{BH}}^2)^{1/2}}, \quad (4)$$

where f_{BH} is a mass ratio of the SBH to the galaxy, and a_{BH} is a softening parameter.

Unit mass, length, and time are 3×10^{10} M_⊙, 5 kpc, and 1.75×10^8 yr, respectively. We chose $a = 0.4$, and $c^2/a = 48.7$, for which the rotational period at $R = 1.0$ is 1.0, and $\epsilon_0 = 0.1$, and $a_{\text{BH}} = 0.01$. Free parameters of this model are f_{BH} and Ω_{bar} , the pattern speed of the bar. We chose $f_{\text{BH}} = 0.01, 0.001$ and 0.0, and $\Omega_{\text{bar}} = 7.0, 3.5$ and 1.0. Typical values for f_{BH} and Ω_{bar} are 0.01 and 3.5, respectively. We summarise these parameters used in our four models in Table 1.

The inner Lindblad resonances (ILRs) exist where

$$\Omega_{\text{bar}} = \Omega(R) - \frac{\kappa(R)}{2}, \quad (5)$$

where Ω is the circular frequency and κ is the epicycle frequency given by

$$\kappa^2 = R \frac{d\Omega^2}{dR} + 4\Omega^2. \quad (6)$$

Fig. 1 shows the radial change of $\Omega - \kappa/2$ for $f_{\text{BH}} = 0.01, 0.001$, and 0.0, and $\Omega_{\text{bar}} = 7.0, 3.5$, and 1.0 are also shown. For $f_{\text{BH}} = 0.0$ (no SBH exists), only the inner ILR exists for $\Omega_{\text{bar}} < 3.5$. For non-zero f_{BH} , the NLR exist at inner region and has larger radius for larger f_{BH} . When $f_{\text{BH}} = 0.01$, the NLR exists for $\Omega_{\text{bar}} > 2.8$, and for $\Omega_{\text{bar}} > 1.2$ when $f_{\text{BH}} = 0.001$. The radii of the NLR and the inner ILR for our four models are shown in Table 1.

2.2 Gas disc model

In the smoothed particle hydrodynamics (SPH) code (Gingold & Monaghan 1982), a gas disc is represented by a large

number of quasi-particles with a certain spatial extent. The SPH particles are randomly distributed within $R = 0.3$ in order to represent uniform density disc at $t = 0$. The initial rotational velocity is given in order to balance centrifugal force caused by the axisymmetric gravitational potential, since the distorted potential is very weak. The rotational period at $R = 0.3$ is 0.3. We assume that gas is isothermal and that its temperature is 10^4 K, which corresponds that the sound speed $C_s \sim 10$ km s⁻¹. Although the effect of the self-gravity of the gas is important for the gas dynamics at the inner region of galaxies, for the first step in investigating the effect of the resonance caused by the central supermassive black hole, we do not take the self-gravity of the gas into account.

2.3 Numerical method

The equation of motion of the i -th SPH particle are

$$\begin{cases} \frac{d\mathbf{r}_i}{dt} = \mathbf{v}_i, \\ \frac{d\mathbf{v}_i}{dt} = -\nabla\Phi_{\text{axi}} - \nabla\Phi_{\text{bar}} - \nabla\Phi_{\text{BH}} - \frac{1}{\rho_i} \nabla(P_i + q_i), \end{cases} \quad (7)$$

where \mathbf{r}_i is the position vector of the i -th particle, and \mathbf{v}_i is velocity vector of the i -th particle, ρ_i , P_i and q_i is density, pressure and artificial viscosity of the gas respectively.

In the SPH method, physical quantities are represented by sum of the smoothing kernel $W(r, h)$. Therefore, the physical quantity f is represented as,

$$f(\mathbf{r}) = \sum_j m \frac{f_j}{\rho_j} W(|\mathbf{r} - \mathbf{r}_j|, h_j), \quad (8)$$

where m is the mass of the SPH particle, and h_i is the particle size. The h_i is varied depending on the local gas density as $h_i \propto \rho_i^{-1/3}$. We chose the Gaussian smoothing kernel,

$$W(r, h) = \frac{e^{-r^2/h^2}}{\pi^{3/2} h^3}. \quad (9)$$

The pressure gradient is represented as a symmetric form

$$\left(\frac{\nabla P}{\rho} \right)_i = \sum_j m \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W(r_{ij}, h_{ij}), \quad (10)$$

where $h_{ij} = (h_i + h_j)/2$. We chose symmetric form because angular momentum is not conserved for the non-symmetric form at our test calculation, and is completely conserved for the symmetric form (see Appendix A).

The artificial viscosity is given by

$$\begin{aligned} \left(\frac{\nabla q}{\rho} \right)_i &= \sum_j (-\alpha \mu_{ij} + \beta \mu_{ij}^2) m \\ &\times \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W(r_{ij}, h_{ij}), \end{aligned} \quad (11)$$

where

$$\mu_{ij} = \frac{2h_{ij}\mathbf{v}_{ij} \cdot \mathbf{r}_{ij}}{C_s(r_{ij}^2 + 0.1h_{ij}^2)}, \quad \alpha = 0.5, \quad \beta = 1.0. \quad (12)$$

We use 10^4 SPH particles. Test calculations of our SPH code are summarised in Appendix B.

3 RESULTS

In this section, we show the time evolution of the gas distribution in our simulations comparing models with and without a central supermassive black hole (SBH).

3.1 A Model without SBH

In order to emphasize the importance of the nuclear Lindblad resonance (NLR) caused by the SBH, we first present results of a model without SBH (model N). The time evolution of the gas disc is as follows (Fig. 2). At first, the gas disc is distorted in a direction leading the bar major axis by about 45° , and two leading spiral shocks are formed at $t = 0.3$. Then, the elongated gas disc begins to rotate clockwise (i.e. retrograde), and becomes parallel to the bar at $t = 0.5$. Since the elongated disc continues to rotate clockwise, the disc is finally led by the bar. In this phase the gas disc gain angular momentum from the bar, and finally it becomes almost axisymmetric. The morphology of the gas disc at $t = 1.0$ is almost the same as that of initial condition. Same as Wada & Habe (1992), no fuelling toward the centre occurs in this model. The time evolution of the outer shape of the gas disc described above is consistent with the adiabatic invariant analysis of a particle orbit in a barred potential (Lynden-Bell 1979).

Fig. 7 & 8 show the time evolution of the total angular momentum of gas disc and of the gas mass within the radius 0.1 for all models. The gas disc of model N loses angular momentum by the torque from the bar, when the distorted gas disc leads the bar ($t < 0.5$). However, after $t = 0.5$ the distorted gas disc becomes to be led by the bar, that is, the gas receives angular momentum from the bar. As a result, the total angular momentum recovers the nearly initial value by the time 1.0. The gas mass within radius 0.1 increases by the effect of the distortion of the gas disc, then it returns to the almost initial value when the gas disc returns to be axisymmetric.

3.2 Models with SBH

3.2.1 Model B: $\Omega_{\text{bar}} = 3.5, f_{\text{BH}} = 0.01$

In this model, $f_{\text{BH}} = 0.01$, which means that the mass of the SBH is about 1 per cent of the mass of a galaxy, and $\Omega_{\text{bar}} = 3.5$ which is an upper limit value for the usual ILR to exist (Fig. 1). These values for f_{BH} and Ω_{bar} are typical for galaxies. The NLR is located at $R = 0.14$. Fig. 3 shows the time evolution of the gas disc in this model. The evolution of the gas disc at $t < 0.2$ is almost the same as the model without SBH. However, at $t = 0.2$, two trailing spiral shocks which lead the bar about 45° occur, and they extend to the edge of the gas disc. Then, the gas loses angular momentum substantially at the shocks and it flows drastically into the centre to make a gas ring with a radius of 0.05. At $t = 0.6$, the elongated gas disc becomes parallel to the bar, then almost all the gas are fuelled to the central gas ring.

The total angular momentum of gas disc are reduced to about 10 per cent of the initial value by the time 0.5 (Fig. 7). On the nearly equal time scale, almost all the gas falls into the central gas ring (Fig. 8).

3.2.2 Model Ba: $\Omega_{\text{bar}} = 3.5, f_{\text{BH}} = 0.001$

For comparison, we simulate the model with the smaller SBH ($f_{\text{BH}} = 0.001$) In this model, the NLR is at $R = 0.06$, which is about half of that of model B. Time evolution of the outer region of the gas disc ($R > 0.1$) is almost the same as model N (Fig. 4). This means that the SBH does not affect the outer region of the gas disc in this model. Therefore, the total angular momentum of gas disc evolves quite similar with model N (Fig. 7), because the outer region of the gas disc is dominant in the total angular momentum.

However, at the inner region ($R < 0.1$) of the gas disc, small two trailing spiral shocks occur, and the gas in the inner region are fuelled into the centre. The time evolution of the gas mass within $R = 0.1$ is similar to model N (Fig. 8). However, the gas mass within the much inner radius evolves quite different from model N, and the gas dynamics at the inner region is almost the same except for model B with the difference of the spatial scale.

3.2.3 Model Bb: $\Omega_{\text{bar}} = 7.0, f_{\text{BH}} = 0.01$

We calculate the case with the bar rotates so fast that the only NLR exists (Fig. 5). The NLR is located at $R = 0.09$, which is a little smaller than model B. The outer region of the gas disc ($R > 0.2$) evolves like model N, although the distortion of gas disc is weak, and a time scale of the pattern rotation of the distorted gas disc is smaller than model N. Fig. 7 shows this property of outer region. This property is because the angular momentum of outer region of the gas disc is dominant in the total angular momentum. The change in the total angular momentum is smaller than model N (Fig. 7), because the distortion of the gas disc is weaker than model N (Fig. 2 & 5). Time scale of the total angular momentum corresponds to the time scale of the pattern rotation of the distorted disc.

Evolution at the inner region of the gas disc is similar to that in model B (e.g. trailing spiral shocks almost parallel to the bar). Only the gas initially located at inner region ($R < 0.15$) are fuelled into the central gas ring. The radius of the gas ring is almost equal to that of model B. However, the accreted gas mass is small (Fig. 8).

3.2.4 Model Bc: $\Omega_{\text{bar}} = 1.0, f_{\text{BH}} = 0.01$

We calculate the case with the bar rotates much so slow that no resonance exists (Fig. 6). In spite of the absence of the resonance, the time evolution of the gas disc is almost same as model B. Almost all the gas are fuelled into the much inner region than model B, and time evolution of the total angular momentum and the gas mass within the radius of 0.1 are also almost the same with model B (Fig. 7 & 8). However, trailing spiral shocks in this model leads the bar about 60° which is steeper than model B. This property, the slower bar induces steeper shocks and faster bar, is consistent of the results of Wada & Habe (1995) (see Fig. 12 in Wada & Habe 1995).

4 DISCUSSION

4.1 Summary of our results

We have made numerical simulation of the gas discs around a central supermassive black hole (SBH) in a weak barred potential and have shown the effects of the nuclear Lindblad resonance (NLR) caused by the SBH. We find that the gas is highly disturbed by the NLR and trailing spiral shocks are formed and the gas is finally accumulated into the ring around the SBH whose radius is about one-third of that of the NLR. The size of the region occupied by the gas which is accumulated into the gas ring depends on the mass of the SBH or the pattern speed of the bar. In the models with smaller SBH (model Ba) or faster bar (model Bb) than model B, only the gas in the smaller region than model B accretes into the gas ring. In these models, radii of NLR are also smaller than model B. This suggests that the size of gas region occupied by the gas which is accumulated is determined by the radius of NLR. It is notable, however, that the gas fuelling occurs even if the NLR does not exist (model Bc). Wada (1994) has shown that in such a case, due to the dissipative nature of gas, highly disturbed motion can be excited. Therefore, we can explain the strong spiral shocks and, as a result, the rapid gas fuelling by this process in model Bc.

4.2 How does the gas fuelling occur ?

Fig. 9a shows the time evolution of the specific angular momentum of an SPH particle and a test particle (i.e. pressureless and viscosityless) initially located near the NLR in model B. Fig. 10 shows the orbits of these particles. Fig. 9b and 10b are the same as Fig. 9a & 10a, but for model N.

In model N, the time evolution of the specific angular momentum and the orbit of the SPH particle are quite similar to that of the test particle (Fig. 9b & 10b), and the reduction of their angular momentum is small. This is because the NLR dose not exist. Therefore, their orbits are not so distorted and the leading spiral shocks in this model is weak. Therefore, angular momentum and energy reduction of the SPH particle at the shocks is small and the gas fuelling dose not occur.

In model B, both the SPH particle and the test particle loses specific angular momentum sufficiently (Fig. 9a), and they take very distorted orbit (Fig. 10a). Therefore, the trailing spiral shocks in this model are very strong. By the dissipative nature of the gas, the SPH particle loses angular momentum and energy at the shocks and then the orbit of the SPH particle bends abruptly at the shocks. The time and location of the shock passage of the SPH particle are indicated by arrows in Fig. 9a & 10a. Then, the SPH particle descends to a lower angular momentum and lower energy state and, finally, it settles into an almost circular orbit near the SBH. This means, for whole gas disc, that the gas fuelling into the nuclear gas ring occurs. The test particle takes chaotic orbit in this model (Fig. 10a), which is the same result of Hasan & Norman (1990).

4.3 The effect of the sound speed of gas

Englmaier & Gerhard (1997) investigated stationary gas flows in a fixed barred potential, which is similar to our model potential except for the existence of SBH, and found

that the structure of gas flow has two regimes depending on the sound speed of gas. For the lower sound speed gas, off-axis shock flow forms and the gas streams inwards through this shock and forms a gas ring. At the higher sound speed the gas arranges itself in on-axis shock flow pattern and no gas rings are formed. The critical effective sound speed dividing the two regimes is around $\sim 20 \text{ km s}^{-1}$ in their standard model potential and is in the range of values observed in the Milky Way.

Although their results are for the normal ILRs (not NLR), the behaviour of the gas discs in our models with SBH, which are off-axis shocks and resultant gas ring, are very similar to that of the lower sound speed regime of them. This is a reasonable result, because the sound speed of our model is 10 km s^{-1} , and gas responses for the NLR and the OILR are expected to be similar in a linear analysis (Wada 1994). It should be noticed that the size of the gas ring formed in our models is much smaller than that in their models due to NLR.

4.4 Observations

In our simulations, the nuclear gas ring and the spiral shocks associated with the ring are formed. Similar gas distribution is observed in nuclear region of many active galaxies and starburst galaxies (Kenny 1993 for a review). For example, NGC 4314 has a molecular ring with a radius of about 250 pc and a star forming region with similar size (Combes et al. 1992). Mini spiral structures are also observed in this galaxy. These spiral structures are due to extinction by dust patches (Benedict 1980). In NGC 1097 which is a weak Seyfert 2 galaxy with barred spiral structure, there is a molecular ring with the radius of about 700 pc (Gerin, Nakai & Combes 1988). This galaxy has hot spots. IC 342 harbours a barlike molecular gas structure ~ 500 pc in extent (Ishizuki et al. 1990), and the molecular gas structure is very similar to the trailing spiral shocks in our models. IC 342 also has a modest nuclear starburst of 70 pc. In our numerical results, in the gas accumulation phase into the gas ring, violent gas motion is excited and strong shocks occur in the nuclear region. Since massive gas concentrates in these region, strong star formation is expected during this phase. Our numerical results can explain star formation activity in these galaxies.

The trailing spiral shocks formed in our models are short-lived. Therefore, the observed gas distribution which is similar to that of our models is expected to be transient structure whose lifetime is about 10^8 yr .

4.5 Self-gravity of the gas discs

We estimate the self-gravity of the finally formed gas ring in order to investigate whether the gas ring is gravitationally unstable or not, since we do not take into account the self-gravity of the gas in our simulations. If the gas ring is unstable, the gas ring fragments into clumps and can collide each other. Then gas fuelling to more inner region will occur (Wada & Habe 1992, Elmegreen 1994).

We use Toomre's Q value as the indicator of self-gravitational instability and take the gas ring formed in

model B. Toomre's Q value of the gas ring is

$$Q \sim 4 \times 10^{-3} \frac{\kappa(r_{\text{ring}})}{\sigma_{\text{ring}}} \left(\frac{C_s}{10 \text{ km s}^{-1}} \right) = \frac{0.3}{\sigma_{\text{ring}}} \left(\frac{C_s}{10 \text{ km s}^{-1}} \right), \quad (13)$$

where σ_{ring} is the surface density of the gas ring and the radius of the gas ring $r_{\text{ring}} = 0.05$. Therefore, if $\sigma_{\text{ring}} \gtrsim 0.3$, $Q \lesssim 1$ and the gas ring is expected to be unstable. Since the surface density of the gas ring is about 100 times of initial gas disc, the condition of gravitational instability is $\sigma_{\text{initial}} \gtrsim 0.003$ for initial gas disc. In real units, this condition is that the mass of the initial gas disc of 1.5 kpc is larger than about $10^7 M_\odot$. Therefore, the gas ring formed from the gas in a normal galaxy can be gravitationally unstable, and the self-gravity of the gas ring plays a very important role in the time evolution of gas ring.

Finally we should note that the self-gravity of the gas at a galaxy centre and the central SBH play basically the same role on the profile of $\Omega - \kappa/2$. This is clear from the similarity between Fig. 1 in Wada & Habe (1995) and Fig. 1 in this paper. Therefore it is reasonable that the gaseous evolution and the fuelling process in the both papers are similar. Self-gravitational effects on a gas disc in a weak bar with a central SBH should be studied.

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APPENDIX A: THE EFFECT OF SYMMETRISED FORMULATION

We test our SPH code from the point of view of angular momentum conservation. We compare two types of the formulation of the pressure gradient and the artificial viscosity term. The first one is the non-symmetrised type,

$$\left(\frac{\nabla P}{\rho} \right)_i = \sum_j m \frac{P_j}{\rho_i \rho_j} \nabla_i W(r_{ij}, h_{ij}), \quad (A1)$$

which is simply derived from the basic idea of the SPH (Lucy 1977), and the second one is the symmetric type,

$$\left(\frac{\nabla P}{\rho} \right)_i = \sum_j m \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W(r_{ij}, h_{ij}), \quad (A2)$$

We calculate model N by using these two types of pressure-viscosity term and compare the numerical results. In Fig. 11, We show the numerical results of model N with the non-symmetrised code. Until $t \sim 0.3$, distribution of SPH particles is almost the same with the numerical result of model N with the symmetrised code. However, after $t \sim 0.7$ evolution of the gas in the non-symmetrised code is physically incorrect: the dense region becomes unstable and turns to irregular shape. On the other hand, in the numerical results of model N with symmetric type the smooth elongated disc is formed in the same stage. After the stage, the evolution of gaseous structure is completely different between the non-symmetrised code and the symmetrised code.

In Fig. 12, we show the time evolution of the total angular momentum of the gas disc of both types. Until $t \sim 0.4$, the time evolution of the total angular momentum is almost the same in both cases. However, after $t \sim 0.4$ the total angular momentum begins to increase smoothly in the symmetric type code. On the other hand, total angular momentum in the non-symmetrised code weakly increase with vibration.

To see the reason of the difference between the codes, we perform the total angular momentum conservation in a axi-symmetric potential (i.e. without the bar). In this test, we take initial gas discs from the numerical results of the non-symmetrised code in the barred potential: gas discs at $t = 0.2, 0.3, 0.4$, and 0.5. Then we calculate the time evolution of the gas discs without the bar using the symmetrised and non-symmetrised code. The results of the test are shown in Fig. 11; dotted lines are the results of the non-symmetrised code and dashed lines are that of the symmetrised code. We found the total angular momentum conserves completely for all cases with the symmetrised code. However, the total angular momentum in the non-symmetrised code slightly decrease for the calculations from the initial gas discs at $t = 0.2$ and 0.3, and it is substantially reduced for the calculations from the initial gas discs at $t = 0.4$ and 0.5. The time $t = 0.2$ and 0.3 corresponds to

the phase when the shock regions are small, and the time $t = 0.4$ and 0.5 corresponds to the phase when the shock regions extend to the edge of the disc. Therefore, the difference of the total angular momentum reduction for the non-symmetrised code comes from the difference of the extent of shock region. We conclude that the non-symmetrised code cannot correctly deal with the shock with high Mach number. Thus gas dynamics in a rotating weak barred potential is an example that the symmetrised SPH code must be used.

APPENDIX B: THE EFFECT OF NUMERICAL SHEAR VISCOSITY

The artificial viscosity adopted in our SPH code is intend to mimic the bulk viscosity. However, shear viscosity appears implicitly as pointed by Hernquist & Katz (1989). Since the strong shear flows appear in our numerical results, we have made two critical tests whether the artificial viscosity acts as a shear viscosity in such a strong shear flow and the angular momentum is transported artificially. The first test is the comparison of the time evolution of the gas flows by using our SPH code and a mesh code, and the second one is the calculation with much larger number of SPH particles.

We calculate model B with AUSM code (Liou & Steffen 1993; Radespiel & Kroll 1995), and compared the result with that of SPH code. The special merits of AUSM compared to the other upwind schemes are low computational complexity and low numerical diffusion. Fig. 13 shows the velocity fields obtained by using the both code at $t = 0.6$, when the gas fuelling occurs most drastically. The velocity field of the SPH code in Fig. 13 is calculated on a regular grid, using the SPH smoothing algorithm. We also show the time evolution of gas mass within $R = 0.1$ for both code in Fig. 14. Both figures show that the two velocity fields and the time evolution of the gas mass are in good agreement. Therefore, numerical shear viscosity does not affect the gas dynamics in our models.

We have also performed the calculation of model B with 5×10^4 SPH particles (five times larger than our original calculations). The decrease in the smoothing lengths will reduce the effects of the artificial transport of angular momentum. The result of this test is shown in Fig. 14. We compare the time evolution of gas mass within $R = 0.1$ of model B with 10^5 with the result with 10^4 particles in Fig. 14. The difference between two cases is adequately small. This result supports that the smoothing length in the calculation with 10^4 particles is small enough to avoid the artificial shear viscosity.

Although the mesh code which has less numerical diffusion compared with SPH code, we have chosen SPH code for this work. This is because high numerical resolution is naturally achieved in denser region by using SPH, and we have a plan to develop our study into the gas dynamics involving three dimensional motion and self-gravity of gas, and the SPH code is easily extended for the numerical calculation of such a situation.

Table 1. Model parameters. R_{NLR} is the radius of the NLR and R_{IILR} is the radius of the inner ILR.

Model	Ω_{bar}	f_{BH}	R_{NLR}	R_{IILR}
N	3.5	0.0	not exist	0.43
B	3.5	0.01	0.14	0.37
Ba	3.5	0.001	0.06	0.43
Bb	7.0	0.01	0.09	not exist
Bc	1.0	0.01	not exist	not exist

Figure 1. $\Omega(R) - \kappa(R)/2$ in our models. A thick solid line, a thin solid line, and a dashed line are for $f_{\text{BH}} = 0.01, 0.001$, and 0.0, respectively. Three horizontal dotted lines represents the pattern speed of the bars in our models.

Figure 2. Time evolution of the distribution of the SPH particles (left) and the velocity field (right) of gas in model N in the rotating bar coordinates. The bar major axis is fixed horizontally on each frame. Time is represented by the number at right up of each figure. A vector written at right bottom of the figure represents the velocity of 5.0 in out units.

Figure 3. As Fig. 2, but for model B.

Figure 4. As Fig. 2, but for model Ba. Only gas distribution is plotted

Figure 5. As Fig. 2, but for model Bb. Only gas distribution is plotted.

Figure 6. As Fig. 2, but for model Bc. Only gas distribution is plotted.

Figure 7. Time variation of the total angular momentum of gas for each model.

Figure 8. Time variation of the gas mass in $R < 0.1$. Total gas mass in each model is 1.0.

Figure 9. (a) The time evolution of the angular momentum of an SPH particle initially located near the NLR in model B and a test particle of the same initial condition. (b) As (a) but for model N. Arrows in Fig. 9a indicates the time when the SPH particle passes shock region

Figure 10. The orbits of the SPH particle and the test particle of Fig. 9 in the rotating bar coordinates. The bar major axis coincides with the horizontal axis. (a) for model B. (b) for model N. Arrows in Fig. 10a indicates the location where the SPH particle passes shock region, and they correspond to the arrows in Fig. 9a

Figure 11. Time evolution of the distribution of the SPH particles in model N calculated by using the non-symmetrised SPH code.

Figure 12. The time evolution of the total angular momentum of the gas disc. The thick solid line shows the result with the symmetrised code, and the thin solid line shows the result with the non-symmetrised code. Dashed lines and dotted lines are the results of the total angular momentum conservation test. Dashed lines show the results with the symmetrised code, and the dotted lines show the results with the non-symmetrised code (see Appendix A for details of the test).

Figure 13. The velocity fields at $t = 0.6$ obtained by using the AUSM code and SPH code.

Figure 14. Time evolution of the gas mass within $R = 0.1$ in model B calculated by using our SPH code with 10^4 and 5×10^4 SPH particles and AUSM code. Total gas mass in each model is 1.0.



































